Maths II - Question Papers

Winter 2016 17216

16117

3 Hours / 100 Marks Seat No. [ ][ ][ ][ ][ ][ ][ ][ ]

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any <u>TEN</u> of the following: 20

a) If $(a - 2bi) + (b - 3ai) = 5 + 2i$ Find a and b.

b) Express in the form $x + iy$, $\dfrac{(2+i)^2}{2+3i}$

where $x, y \in \mathrm{R}$ and $i = \sqrt{-1}$

c) If $f(x) = x^4 - 2x + 7$ find $f(0) + f(2)$.

d) If $f(x) = 16^x + \log_2^x$ Find the value of $f(1/4)$

e) Evaluate $\lim\limits_{x \to 1} \left(\dfrac{x^3 - 1}{x - 1} \right)$

f) Evaluate $\lim\limits_{x \to 0} \left(\dfrac{3\sin x + 4x}{7x - 2\tan x} \right)$

g) Evaluate $\lim\limits_{x \to 0} \left(\dfrac{3^x - 2^x}{\sin \pi x} \right)$

h) If $y = e^{7x} . \cos 7x$ Find $\dfrac{dy}{dx}$.

P.T.O.

Marks

i) If $y = \log(x.\sin 2x)$ Find $\frac{dy}{dx}$.

j) If $x = 3\sin 4\theta$, $y = 4\cos 3\theta$ Find $\frac{dy}{dx}$.

k) Show that there exist a root of the equation $x^3 - 5x - 11 = 0$ between 2 and 3.

l) Solve the following equations by using Jacobi's method (only first iteration)
$4x - y + z = 4$, $x + 6y + 2z = 9$, $-x - 2y + 5z = 2$

2. Attempt any FOUR of the following: 16

a) Express $(1 + i)$ in polar form.

b) Simplify using De Moiver's theorem.

$$\frac{(\cos 2\theta + j\sin 2\theta)^{3/2}.(\cos\theta - j\sin\theta)^3}{(\cos 3\theta - j\sin 3\theta)^2.(\cos 5\theta - j\sin 5\theta)^{2/5}}$$

c) Use De Moiver's theorem to solve $x^3 - 1 = 0$

d) Separate into real and imaginary parts of $\cosh(\alpha + i\beta)$.

e) If $f(x) = \frac{1}{1-x}$ show that $f\left[f\{f(x)\}\right] = x$.

f) If $f(x) = x^2 - 3x + 4$. Find x if $f(1-x) = f(2x+1)$.

3. Attempt any FOUR of the following: 16

a) If $y = f(x) = \frac{2x-3}{3x-2}$ show that $x = f(y)$.

b) Evaluate: $\lim\limits_{x \to 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$

c) Evaluate: $\lim\limits_{x \to 2} \left(\frac{x}{x-2} - \frac{4}{x^2 - 2x}\right)$

d) Evaluate: $\lim\limits_{\theta \to 0} \frac{2\sin\theta - \sin 2\theta}{\theta^3}$

e) Evaluate: $\lim\limits_{x \to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$

f) Evaluate: $f(x) = \log\left(\frac{1+x}{1-x}\right)$

Prove that: $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$

Marks

4. Attempt any FOUR of the following: 16

a) Using First principle of derivative find derivative of $f(x) = \cos x$.

b) If u and v are differentiable functions of x then prove that

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

c) If $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$ find $\frac{dy}{dx}$.

d) If $4x + 3y = \log(4x - 3y)$ find $\frac{dy}{dx}$.

e) If $x^3 . y^2 = (x + y)^5$ show that $\frac{dy}{dx} = y/x$.

f) If $x = a(2\theta - \sin 2\theta)$, $y = a(1 - \cos 2\theta)$

Find $\frac{dy}{dx}$ at $\theta = \pi/4$.

5. Attempt any FOUR of the following: 16

a) Evaluate $\lim\limits_{x \to 0} \frac{(5^x - 1)\tan x}{\sqrt{x^2 + 16} - 4}$

b) Evaluate $\lim\limits_{x \to 3} \frac{\log x - \log 3}{x - 3}$

c) Using Bisection method find the approximate root of the equation $x^3 - 5x + 1 = 0$ (three iterations only)

d) Find the root of $x^2 - x - 1 = 0$ by using Regula Falsi Method up to third approximation.

e) Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method upto three iterations only.

f) Use Newton-Raphson method to find $\sqrt[3]{20}$ correct to three decimal places. (third iteration)

Marks

6. Attempt any <u>FOUR</u> of the following: 16

a) If $y = (x + \sqrt{x^2 + 1})^m$ show that $(x^2 + 1)\frac{d^2 y}{dx^2} + x\frac{dy}{dx} - m^2 y = 0$

b) If $x = 2\cos t - \cos 2t,\ y = 2\sin t - \sin 2t$ Find $\frac{d^2 y}{dx^2}$ at $t = \pi/2$.

c) Solve the following equations by Gauss-elimination method.

$x + 2y + 3z = 14,\ 3x + 3y + 5z = 24$

$4x + 5y + 7z = 35.$

d) Solve the following equations by Jacobi's method (take three iterations)

$5x + 2y + z = 12,\ x + 4y + 2z = 15,\ x + 2y + 5z = 20$

e) Solve the equations by Gauss-seidal method upto two iterations.

$10x + 2y + z = 9,\ x + 10y - z = -22,\ -2x + 3y + 10z = 22.$

f) Solve the following equations by Jacobi's method (Take three iterations

$10x + y + z = 12$

$x + 10y + z = 12$

$x + y + 10z = 12$

Summer 2016 17216

15162

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Illustrate your answers with neat sketches wherever necessary.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

Marks

1. Attempt any <u>TEN</u> of the following: 20

a) If $3a-7+2bi=5i+ia-5b$ find a, b.

b) If $z=1+i\sqrt{3}$ show that $z^2+4=2z$.

c) Define even and odd function.

d) If $f(x)=\sin x$ show that $f(3x)=3f(x)-4f^3(x)$

e) Evaluate $\lim\limits_{x\to 1}\left(\dfrac{1}{x-1}-\dfrac{1}{x^2-x}\right)$

f) Evaluate $\lim\limits_{x\to 0}\dfrac{\cos 5x-\cos 3x}{x^2}$

g) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{x}}$

h) Find $\dfrac{dy}{dx}$ if $y=\dfrac{\sin x}{1-\cos x}$

i) If $y=\log(\sec x+\tan x)$ find $\dfrac{dy}{dx}$

j) If $\tan^{-1}(x^2+y^2)=a^2$ Find $\dfrac{dy}{dx}$

Marks

k) Using Bisection method find the root of $x^3 - x - 1 = 0$ (two iteration only)

l) Find by Jacobis method, the first iteration only, for the following equation $5x - y = 9, x - 5y + z = -4, y - 5z = 6$.

2. Attempt any FOUR of the following: **16**

a) Find the complex conjugate of $\frac{(2+i)^2}{2+3i}$

b) Simplify using De-moiver's theorem

$$\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 5\theta - i\sin 5\theta)^{\frac{4}{5}}}{\left(\cos\frac{9}{2}\theta + i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}}\left(\cos\frac{4}{5}\theta - i\sin\frac{4}{5}\theta\right)^{10}}$$

c) Using Euler's Exponential formula prove that:

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\cos h^2\theta - \sin h^2\theta = 1$

d) Use De-moivre's theorem to solve the equation $x^3 - 1 = 0$

e) If $f(x) = \frac{x+3}{4x-5}$ and $t = \frac{3+5x}{4x-1}$ then show that $f(t) = x$

f) If $f(t) = 50\sin(50\pi t + 0.04)$ show that $f\left(\frac{2}{100} + t\right) = f(t)$

3. Attempt any FOUR of the following: **16**

a) If $f(x) = x^2 + 3$ then find the value of x for which $f(x) = f(2x+1)$

b) If $f(x) = 16^x + \log_2^x$ then find the value of $f\left(\frac{1}{4}\right)^2, f\left(\frac{1}{2}\right)$

c) Evaluate $\lim_{x \to 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$

d) Evaluate $\lim_{x \to 3} \frac{\log x - \log 3}{x - 3}$

e) Evaluate $\lim_{x \to 0} \frac{4^x + 4^{-x} - 2}{x \cdot \sin x}$

f) Evaluate $\lim_{\theta \to \frac{\pi}{4}} \frac{\sin\theta - \cos\theta}{\theta - \frac{\pi}{4}}$

Marks

4. Attempt any FOUR of the following: 16

a) Using first principle find the derivative of $f(x) = x^n, x \in R$.

b) If u and v are differentiable functions of x and if $y = uv$ then prove that: $\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{dy}{dx}$

c) Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{2x}{1+35x^2}\right)$

d) If $x^3y^2 = (x+y)^5$ show that $\frac{dy}{dx} = \frac{y}{x}$

e) If $y = \frac{(1-x)^{\frac{1}{2}}}{(x-1)^{\frac{5}{7}}(2x+1)^{\frac{1}{3}}}$ find $\frac{dy}{dx}$

f) Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$

5. Attempt any FOUR of the following: 16

a) Evaluate $\lim\limits_{x \to \infty} \sqrt{x}\left(\sqrt{x+1} - \sqrt{x}\right)$

b) Evaluate $\lim\limits_{\theta \to \frac{\pi}{4}} \frac{2-\sec^2\theta}{1-\tan\theta}$

c) Using bisection method find the approximate value of $\sqrt{10}$ by performing three iterations.

d) Using Regula Falsi method find the root of $x^2 - \log_{10}^{x} = 12$ (upto three iterations only)

e) Find the approximate root of the equation $x^3 - 20 = 0$ by Newton-Raphson method (three iterations)

f) Obtain the approximate root value of equation $x^3 - 4x + 1 = 0$ using Regula-Falsi method upto 4 decimal places.

Marks

6. Attempt any FOUR of the following: 16

a) Differentiate $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

b) If $y = e^{\tan^{-1}x}$ show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$

c) Solve using Gauss-Elimination method

$x + 2y + 3z = 14$

$3x + y + 2z = 11$

$2x + 3y + z = 11$

d) Solve by Gauss Seidal method (upto the iterations any)

$x + 7y - 3z = -22$

$5x - 2y + 3z = 18$

$2x - y + 6z = 22$

e) Solve the equations using Jacobi's method (upto three iterations)

$10x - 2y - 2z = 6$

$-x - y + 10z = 8$

$-x + 10y - 2z = 7$

f) Use Gauss-Seidal method to solve following equations (use two iterations)

$10x + 2y + z = 9$

$x + 10y - z = -22$

$-2x + 3y + 10z = 22$

Winter 2015

17216

15116

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

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(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any <u>TEN</u> of the following: 20

a) If $z = 1 + 3i$ evaluate $z^2 + 2z + 4$

b) Express $1 + i$ in modulus and amplitude form.

c) If $f(x) = 16^x + \log_4 x$ find $f\left(1/2\right)$.

d) Define even and odd function.

e) Evaluate $\lim\limits_{x \to 1} \dfrac{x^2 + 2x + 5}{x + 1}$

P.T.O.

Marks

f) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\tan 5x}$

g) Evaluate $\lim\limits_{x \to 0} \dfrac{3^{2x} - 2^{3x}}{\sin x}$

h) If $y = e^{4x} \cos 3x$ find $\dfrac{dy}{dx}$

i) If $y = \log [\sin (4x - 3)]$ find $\dfrac{dy}{dx}$

j) Find $\dfrac{dy}{dx}$ if $x = 4 \sin 3\theta, y = 4 \cos 6\theta$

k) Show that the root of $x^3 - 9x + 1 = 0$ lies between 2 and 3.

l) Find the first iteration by using Jacobi's method for the following system of equations:

$5x + 2y + z = 12, \quad x + 4y + 2z = 15, \quad x + 2y + 5z = 20$

2. Attempt any <u>FOUR</u> of the following: **16**

a) Find cube roots of unity and show that one root is square the other.

b) Simplify : $\dfrac{(\cos 2\theta + i \sin 2\theta)(\cos \theta - i \sin \theta)^4}{(\cos 3\theta + i \sin 3\theta)(\cos 5\theta - i \sin 5\theta)^3}$

using De-Moiver's theorem.

c) If $\sin (A + iB) = x + iy$ prove that:

(i) $\dfrac{x^2}{\cos h^2 B} + \dfrac{y^2}{\sin h^2 B} = 1$

(ii) $\dfrac{x^2}{\sin^2 A} - \dfrac{y^2}{\cos^2 A} = 1$

Marks

d) Using Euler's exponential formula prove that

$\sin^2\theta + \cos^2\theta = 1$

e) If $f(x) = \log\left(\frac{x}{x-1}\right)$ show that $f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right)$

f) If $f(x) = \frac{3x+2}{4x-3}$ show that $f = f^{-1}$

3. Attempt any <u>FOUR</u> of the following: 16

a) If $f(x) = \frac{x+3}{4x-5}$ and $t = \frac{3+5x}{4x-1}$ show that $f(t) = x$.

b) If $f(t) = 50\sin(100\,\pi t + 0.04)$, then show that $f\left(\frac{2}{100} + t\right) = f(t)$

c) Evaluate $\lim\limits_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$

d) Evaluate $\lim\limits_{x \to 0} \frac{\sin 2x - 2\sin x}{x^3}$

e) Evaluate $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cos 3x + 3\cos x}{\left(\pi/2 - x\right)^3}$

f) Evaluate $\lim\limits_{x \to 3} \frac{\log(x-2)}{x^2 - 9}$

Marks

4. Attempt any FOUR of the following: **16**

a) If u and v are differentiable functions of x and $y = u/v$ where $v \neq 0$ then prove that $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$.

b) By using first principle find the derivative of $y = \cos x$.

c) If $y = \sin^{-1}\left[\frac{1}{\sqrt{1+x^2}}\right]$ find $\frac{dy}{dx}$

d) Find $\frac{dy}{dx}$ if $y = \frac{(\cos x)^x}{(1+x^2)}$

e) If $x^p \cdot y^q = (x+y)^{p+q}$ show that $\frac{dy}{dx} = \frac{y}{x}$

f) If $y = 3\sin t - 2\sin^3 t$, $x = 3\cos t - 2\cos^3 t$ find $\frac{dy}{dx}$ at $t = \pi/4$.

5. Attempt any FOUR of the following: **16**

a) Evaluate $\lim\limits_{x \to 0} \left(\frac{6^x - 3^x - 2^x + 1}{x^2}\right)$

b) Evaluate $\lim\limits_{x \to 3} \frac{\log x - \log 3}{x - 3}$

c) Find the approximate roots of the equation $x^3 - x - 4 = 0$ by Bisection method.

Marks

d) Show that root of the equation $x^3 - 4x + 1 = 0$ in (1, 2) and find it by using Newton-Raphson method performing two iterations.

e) Solve the following equations by Gauss elimination method.

$x + 2y + 3z = 14, \quad 3x + y + 2z = 11, \quad 2x + 3y + z = 11.$

f) Solve the following equations by Gauss-Seidal method.

$5x - y = 9, \quad x - 5y + z = -4, y - 5z = 6$

6. Attempt any FOUR of the following: 16

a) If $y = e^{m \sin^{-1} x}$ prove that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - m^2 y = 0$

b) If $x = a(\theta + \sin\theta), \quad y = a(1 + \cos\theta)$ find $\dfrac{d^2y}{dx^2}$ at $\theta = \pi/2$.

c) Obtain the root of the equation by Regula-Falsi method.
$x^3 - x - 1 = 0$

d) Solve the following equation by Jacobi's method
$20x + y - 2z = 17; \ 3x + 20y - z = -18; \ 2x - 3y + 20z = 25.$

e) Solve the equation by using Gauss - eliminaion method.
$4x + y + 2z = 12, \ -x + 11y + 4z = 33, \quad 2x - 3y + 8z = 20$

f) Use Newton-Raphson method to evaluate $\sqrt[3]{20}$ correct to three decimal places.

Summer 2015

17216

21415

3 Hours / 100 Marks Seat No.

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Figures to the right indicate full marks.

(4) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any <u>TEN</u> of the following: 20

a) If $\frac{10}{3+4i} = a + ib$ find a and b.

b) If $z = 3 + 4i$ Find $z^2 - 6z + 25$

c) If $f(x) = x^2 + 6x + 10$ Find $f(2) + f(-2)$

d) If $f(x) = \frac{a^x + a^{-x}}{2}$ prove that the function is even function.

e) Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

f) Evaluate $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

g) Evaluate $\lim_{x \to \infty} \left(\frac{x}{x+1}\right)^x$

h) If $y = e^x \sin x$ find $\frac{dy}{dx}$.

Marks

i) If $y = \tan^{-1}\left(\frac{a+x}{1-ax}\right)$ Find $\frac{dy}{dx}$.

j) If $x = a \sec t$ and $y = b \tan t$ then find dy/dx.

k) Prove that the root of equation $x^3 - x - 4 = 0$ lies between 0 and 2.

l) Find the first iteration by using Jacobi's method for the following equation.

$4x + y + 3z = 17$, $x + 5y + z = 14$ and $2x - y + 8z = 12$.

2. Attempt any <u>FOUR</u> of the following: 16

a) If $f(x) = \tan x$, prove that $f(2x) = \frac{2f(x)}{1-f^2(x)}$

b) Simplify using De-moiver's theorem

$$\frac{(\cos 3\theta + i\sin 3\theta)^4 \ (\cos 5\theta - i\sin 5\theta)^{4/5}}{(\cos {}^3\!/\!{}_5\theta + i\sin {}^3\!/\!{}_5\theta)^5 \ (\cos {}^4\!/\!{}_5\theta + i\sin {}^4\!/\!{}_5\theta)^{10}}$$

c) Seperate into real and imaginary part of $\sin(x + iy)$

d) Express in Polar form $1 - \sqrt{3}\, i$

e) Show that $(1 + i)^{12} + (1 - i)^{12} = -128$

f) If $f(x) = \log\left(\frac{x+1}{x-1}\right)$ prove that $f\left(\frac{1+x^2}{2x}\right) = 2f(x)$

Marks

3. Attempt any FOUR of the following: 16

a) Find $f(t)$, if $f(x)=\dfrac{2x+5}{3x-4}$ and $t=\dfrac{5+4x}{3x-2}$.

b) Evaluate $\lim\limits_{x\to 0}\dfrac{3^x+3^{-x}-2}{x^2}$

c) If $f(x)=x^2-3x+4$ solve $f(1-x)=f(2x+1)$.

d) Evaluate $\lim\limits_{x\to 3}\dfrac{x^3-7x^2+15x-9}{x^3-4x^2-3x+18}$.

e) Evaluate $\lim\limits_{x\to\infty}\sqrt{x^2+x+1}-x$

f) Evaluate $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{x^3}$

4. Attempt any FOUR of the following: 16

a) Find $\dfrac{dy}{dx}$ if $y=\cos^{-1}(2x^2-1)$

b) If $x^2+y^2-xy=0$ find $\dfrac{dy}{dx}$.

c) If $x=a(1+\cos\theta)$ $y=a(1-\cos\theta)$ find $\dfrac{dy}{dx}$.

d) Using first principle find derivate of $f(x)=\tan x$.

e) If u and v are differentiable functions of x and $y=u+v$ than prove that $\dfrac{dy}{dx}=\dfrac{du}{dx}+\dfrac{dv}{dx}$.

f) If $x^y=e^{x-y}$ prove that $\dfrac{dy}{dx}=\dfrac{\log x}{(1+\log x)^2}$.

Marks

5. Attempt any FOUR of the following: 16

a) Evaluate $\lim_{x \to 3} \dfrac{\log x - \log 3}{x-3}$

b) Evaluate $\lim_{x \to 0} \dfrac{(5^x-1)\tan x}{\sqrt{x^2+16}-4}$

c) Find the root of the equation $x^3-9x+1=0$ which lies between 2 and 3 using Regula falsi method.

d) Find a root of $x^3-9x^2-18=0$ by Newton-Raphson method. (carry out 3 iterations)

e) Using Newton-Raphson method, find approximate value of $\sqrt{10}$ (carry out 3 iterations)

f) Find the root of equation $x^3-4x+1=0$ using bisection method (carry out 3 iterations)

6. Attempt any FOUR of the following: 16

a) Find $\dfrac{d^2y}{dx^2}$ if $x = a\cos\theta \quad y = a\sin\theta$.

b) Solve the following equation by Gauss elimination method

$2x+y+z=10,\ 3x+2y+3z=18$ and $x+4y+9z=16$.

c) Solve the following equation by Gauss-Sedial method taking two iterations.

$10x+y+z=12,\ 2x+10y+z=13$ and $2x+2y+10z=14$.

d) Solve the following equation by Jacobi's method by performing two iteration's only

$15x+2y+z=18,\ 2x+20y-3z=19$ and $3x-6y+25z=22$.

e) Solve by Jacobi method, carry out two iterations

$10x+y+2z=13;\ 3x+10y+z=14;\ 2x+3y+10z=15$.

f) If $y = e^{m\sin^{-1}x}$ prove that $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=m^2y=0$

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14115

Winter 2014

17216

3 Hours/100 Marks

Seat No.

Instructions : (1) ***All*** *questions are* ***compulsory****.*

(2) *Answer* ***each*** *next main question on* ***a new*** *page.*

(3) *Figures to the* ***right*** *indicate* ***full*** *marks.*

(4) ***Assume*** *suitable data, if* ***necessary****.*

(5) ***Use*** *of Non-programmable Electronic Pocket Calculator is* ***permissible****.*

(6) *Mobile Phone, Pager and any other Electronic Communication devices are* ***not permissible*** *in Examination Hall.*

MARKS

1. Attempt **any ten** of the following : **20**

a) If $(3x - 4y) + i(x + y) = 7$ find x, y.

b) If $z = 1 + \sqrt{3}\, i$, show that $z^2 + 4 = 2z$.

c) If $f(x) = 3x^2 - 5x + 7$, show that $f(-1) = 3f(1)$.

d) State whether the function $f(x) = \dfrac{e^x + e^{-x}}{2}$ is odd or even.

e) Evaluate $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

f) Evaluate $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$.

g) Evaluate $\lim\limits_{x \to 0} \dfrac{3^x - 4^x}{x}$.

h) Find $\dfrac{dy}{dx}$, if $y = \log(x^2 + 2x)$.

P.T.O.

MARKS

i) If $x^2 + y^2 = 4$, find $\frac{dy}{dx}$.

j) Find $\frac{dy}{dx}$, if $x = \sin\theta$, $y = \cos\theta$.

k) Show that root of equation $x^3 - 2x - 5 = 0$ lies between 2 and 3.

l) Find the first iteration by using Jacobi's method for the following system of equation.

$10x + y + 2z = 13$, $3x + 10y + z = 14$, $2x + 3y + 10z = 15$.

2. Attempt **any four** of the following : **16**

a) Express the following complex number in polar form $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

b) Evaluate $(1 + i)^8 + (1 - i)^8 = 32$.

c) Using Euler's formula prove that $\sin^2\theta + \cos^2\theta = 1$.

d) Simplify using De-Moivres theorem.

$$\frac{(\cos 5\theta - i\sin 5\theta)^{2/5}\ (\cos \tfrac{2}{7}\theta + i\sin \tfrac{2}{7}\theta)^7}{(\cos 4\theta + i\sin 4\theta)^{1/4}\ (\cos \tfrac{2}{3}\theta - i\sin \tfrac{2}{3}\theta)^3}$$

e) If $y = f(x) = \dfrac{2x - 3}{3x - 2}$ then prove that $x = f(y)$.

f) If $f(x) = x^2 - 4x + 11$, solve the equation $f(x) = f(3x - 1)$.

3. Attempt **any four** of the following : **16**

a) If $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$ then prove that $f\left(\dfrac{2x}{1+x^2}\right) = 2\,f(x)$.

b) If $f(x) = \dfrac{1}{1-x}$, show that $f\,[\,f\,\{f(x)\}\,] = x$.

c) Evaluate $\lim\limits_{x \to 1} \dfrac{x^3 + 3x^2 - 6x + 2}{x^3 + 3x^2 - 3x - 1}$.

Marks

d) Evaluate $\lim\limits_{x \to \infty} \left[\sqrt{x^2 + 5x} - x\right]$.

e) Evaluate $\lim\limits_{x \to 0} \dfrac{6^x - 3^x - 2^x + 1}{x^2}$.

f) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x - 3\sin x}{x^3}$.

4. Attempt **any four** of the following : **16**

a) Using first principle find derivative of f(x) = sinx.

b) If u and v are differentiable functions of x and y = u.v, then prove that

$$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}.$$

c) If $y = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$, find $\dfrac{dy}{dx}$

d) Differentiate w.r.t. x, $\tan^{-1}\left(\dfrac{5x}{1 - 6x^2}\right)$.

e) If $y = (\sin x)^{\cos x}$, find $\dfrac{dy}{dx}$.

f) If $y = \tan^{-1}\left(\dfrac{2t}{1 - t^2}\right)$ and $x = \sin^{-1}\left(\dfrac{2t}{1 + t^2}\right)$, find dy/dx.

5. Attempt **any four** of the following : **16**

a) Evaluate $\lim\limits_{x \to 0} \dfrac{\tan x\,(5^x - 1)}{\left(\sqrt{x^2 + 16} - 4\right)}$.

b) Evaluate $\lim\limits_{x \to 3} \dfrac{\log x - \log 3}{(x - 3)}$.

Marks

c) Using Bisection method, find the approximate root of $x^3 - 6x + 3 = 0$ (three iteration only).

d) Using Regula Falsi method, find the root of $x^3 - x - 4 = 0$ (three iteration only).

e) Using Newton-Raphson method, find the root of $x^4 - x - 9 = 0$.

f) Using Newton-Raphson method, find the approximate value of $\sqrt{10}$ (three iteration only).

6. Attempt **any four** of the following : **16**

a) If $y = \sin 5x - 3\cos 5x$, show that $\frac{d^2y}{dx^2} + 25y = 0$.

b) If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

c) Solve by Jacobi's method (three iteration only)

$5x + 2y + 7z = 30$, $x + 4y + 2z = 15$

$x + 2y + 5z = 20$

d) Solve by Gauss elimination method

$x + 2y + 3z = 14$, $3x + y + 2z = 11$

$2x + 3y + z = 11$

e) Solve by Jacobi's method (three iteration only)

$20x + y - 2z = 17$, $3x + 20y - z = -18$

$2x - 3y + 20z = 25$.

f) Solve by Gauss – Seidal method (three iteration only)

$15x + 2y + z = 18$, $2x + 20y - 32 = 19$

$3x - 6y + 25z = 22$.

21314

Summer 2014

17216

3 Hours/100 Marks

Seat No. | | | | | | | | |

Instructions : *(1)* ***All*** *questions are* ***compulsory****.*

(2) Illustrate your answers with neat sketches ***wherever*** *necessary.*

(3) Figures to the ***right*** *indicate* ***full*** *marks.*

(4) Assume suitable data, if ***necessary****.*

(5) Use of Non-programmable Electronic Pocket Calculator is ***permissible****.*

(6) Mobile Phone, Pager and any other Electronic Communication devices are ***not permissible*** *in Examination Hall.*

MARKS

1. Attempt **any ten** of the following : **20**

a) If f(x) = cos x, show that $f(3x) = 4f^3(x) - 3f(x)$.

b) Express in the form a + ib, $\frac{1+i}{2-i}$ where $a, b \in R$, $i = \sqrt{-1}$.

c) Evaluate $\lim\limits_{x \to 0} \frac{1}{\sqrt{x+1}-1}$.

d) Evaluate $\lim\limits_{x \to 0} \frac{2^x - 1}{\sin 2x}$.

e) If $f(x) = 3x^2 - 5x + 7$ show that f(–1) = 3f(1).

f) Find x and y, if x [1 – i] + y [2 + i] + 6 = 0.

g) Evaluate $\lim\limits_{x \to 0} \frac{1-\cos 2x}{x^2}$.

h) If $y = \cos^{-1}(\sin x)$ find $\frac{dy}{dx}$.

i) If $y = e^x \cdot \sin x \cdot \cos x \cdot$ find $\frac{dy}{dx}$.

P.T.O.

Marks

j) Find $\frac{dy}{dx}$ if $y = x^x$.

k) Find first two real roots of equation, $x^3 - 2x - 5 = 0$ using bisection method.

l) Find the first iteration by using Jacobi's method for the following system of equation

$5x - y + z = 10$, $x + 2y = 6$, $x + y + 5z = -1$.

2. Attempt **any four** of the following : **16**

a) If $f(x) = \frac{x-4}{4x-1}$ then show that $f[f(x)] = x$.

b) If $f(x) = \log\left[\frac{1+x}{1-x}\right]$ then show that $f(a) + f(b) = f\left[\frac{a+b}{1+ab}\right]$.

c) Using Euler's formula prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

d) Simplify using De Moivre's theorem :

$$\frac{(\cos 5\theta - i\sin 5\theta)^{2/5} \cdot (\cos \tfrac{2}{7}\theta + i\sin \tfrac{2}{7}\theta)^2}{(\cos 4\theta + i\sin 4\theta)^{1/4}\ [\cos \tfrac{2}{3}\theta - i\sin \tfrac{2}{3}\theta]^3}$$

e) Find cube-roots of unity.

f) Simplify $1 + i^{100} + i^{10} + i^{50}$.

3. Attempt **any four** of the following : **16**

a) If $f(x) = ax^2 + bx + 3$ and $f(1) = 4$, $f(2) = 11$, find 'a' and 'b'.

b) If $f(x) = \sin x$, $g(x) = \cos x$ prove that :

i) $f(x + y) = f(x)\, g(y) + g(x)\, f(y)$.

ii) $g(m - n) = g(m) \cdot g(n) + f(m).f(n)$.

c) Evaluate $\lim\limits_{x \to \pi/4} \frac{\sin^2 x - \cos^2 x}{1 - \tan x}$.

d) Evaluate $\lim\limits_{x \to \infty} x\left[\sqrt{x^2 + 1} - \sqrt{x^2 - 1}\right]$.

MARKS

e) Evaluate $\lim_{x \to 4} \frac{x^2 - 7x + 12}{x^3 - 64}$.

f) Evaluate $\lim_{x \to 0} \frac{a^x + a^{-x} - 2}{\sin^2 x}$.

4. Attempt **any four** of the following : **16**

a) If u and v are differentiable functions of x and $y = u/v$, then prove that

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}.$$

b) If $y = \sin^{-1}[3x - 4x^3]$ find dy/dx.

c) Find $\frac{dy}{dx}$, if $13x^2 + 2x^2y + y^3 = 1$.

d) Find the derivative of $(x) \sin^{-1} x$.

e) Using first principle find derivative of $f(x) = a^x$.

f) Find dy/dx if $y = \log\left[x + \sqrt{x^2 + a^2}\right]$.

5. Attempt **any four** of the following : **16**

a) Evaluate $\lim_{x \to 1} \frac{\sin \pi x}{x - 1}$.

b) Evaluate $\lim_{x \to 0} \frac{x}{\sqrt{9 - x + x^2} - 3}$.

c) Using Bisection method find the approximate root of $x^2 + x - 3 = 0$ [carry out three iteration].

d) Using Newton-Raphson method find approximate value of $\sqrt[3]{100}$ (perform three iterations).

MARKS

e) Using Regula Falsi method find approximate root of equation $x^3 + 2x^2 - 8 = 0$ (take three iteration).

f) Find the real root of the equation $x \cdot e^x = 3$ using False position method (Two iterations only).

6. Attempt **any four** of the following : **16**

a) Differentiate $\cos^{-1}[2x^2 - 1]$ w.r.t. $\sin^{-1}\left[2x \cdot \sqrt{1-x^2}\right]$.

b) If y = sin 5x – 3 cos 5x, show that $\frac{d^2y}{dx^2} + 25y = 0$.

c) Solve the following equations by Jacobi's method, performing three iterations only :

20x + y – 2z = 17, 3x + 20y – z = –18 and 2x – 3y + 20z = 25.

d) Solve the following equations by Gauss-Seidal method taking three iterations only :

15x + 2y + z = 18, 2x + 20y – 3z = 19, 3x – 6y + 25z = 22.

e) Solve the following equations by Gauss elimination method,

x + 2y + 3z = 14, 3x + y + 2z = 11 and 2x + 3y + z = 11.

f) With the following system of equations 5x – y = 9, x – 5y + z = – 4, y – 5z = 6 set up the Gauss-Seidal iterations scheme for solution. Iterate two times, using initial approximations as $x_0 = 1.5$, $y_0 = 0.5$, $z_0 = -0.5$.

———————

Scheme – G

Sample Question Paper

Course Name: All Branches of Diploma in Engineering and Technology

Course Code : AE/CE/CH/CM/CO/CR/CS/CW/DE/EE/EP/IF/EJ/EN/ET/EV/EX/IC/IE/IS/ME/ MU/PG/PT/PS/ CD/CV/ED/EI/FE/IU/MH/MI/AU

Semester : Second

Subject Title : Engineering Mathematics **17216**

Marks : 100 **Time: 3 Hrs.**

Instructions:

1. All questions are compulsory.
2. Illustrate your answers with neat sketches wherever necessary.
3. Figures to the right indicate full marks.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.

Q.1Attempt any TEN of the following **20 Marks**

a) If (3 + i)x + (1 – i)y = 1 + 7i find the value of x & y .

b) Express in the form a+ib, $\frac{2+i}{1-i}$, where a , b $\in$ R , i = $\sqrt{-1}$.

c) IF f(x) = $x^3 - 3x^2 + 5$, find f(0) + f(3) .

d) Define even and odd function.

e) Evaluate $\lim_{x\to 2}\left[\frac{x-2}{\sqrt{x}-\sqrt{2}}\right]$.

f) Evaluate $\lim_{x\to 0}\left[\frac{\tan 5x}{\sin 6x}\right]$.

g) Evaluate $\lim_{x\to 0}\left[\frac{e^{\sin 3x}-1}{2x}\right]$.

h) If y= e^{3x}. sin5x, find $\frac{dy}{dx}$.

i) If y = log[tan(4 – 3x)] , find $\frac{dy}{dx}$.

j) Find $\frac{dy}{dx}$ if x = $3at^2$ and y = $2at^3$.

k) Show that there exist a root of the equation $x^3 - 4x + 1 = 0$ in the interval (1 , 2) .

l) Find the first iteration by using Jacobi's method for the following system of equation

$5x - y = 9$, $x - 5y + z = -4$, $y - 5z = 6$.

Q.2 Attempt any FOUR of the following **16 Marks**

a) Find modulus and argument of $\frac{1}{2}-\frac{\sqrt{3}}{2}i$, hence express in polar form.

b) Simplify using De Moivre's Theorem

$$\frac{(\cos 2\theta+i\sin 2\theta)^3(\cos 3\theta-i\sin 3\theta)^4}{(\cos\theta+i\sin\theta)^2(\cos 2\theta-i\sin 2\theta)^{-3}}$$

c) Use De Moivre's theorem to solve $x^4+1=0$.

d) If $x+iy = \sin(A+iB)$ prove that

i) $\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$ ii) $\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}=1$

e) If $f(x)=\frac{x+2}{4x-3}$ and $t=\frac{2+3x}{4x-1}$. Show that $f(t)=x$.

f) If $f(x)=\tan x$, show that

i) $f(2x)=\frac{2f(x)}{1-[f(x)]^2}$ ii) $f(\alpha+\beta)=\frac{f(\alpha)+f(\beta)}{1-f(\alpha).f(\beta)}$.

Q.3 Attempt any FOUR of the following **16 Marks**

a) If $f(t)=50\sin(100\pi t+0.04)$, show that $f(\frac{2}{100}+t)=f(t)$

b) If $f(x)=\log\left(\frac{1+x}{1-x}\right)$, prove that $f\left(\frac{2x}{1+x^2}\right)=2f(x)$

c) Evaluate $\lim_{x\to 3}\left[\frac{1}{x-3}-\frac{3}{x^3-5x^2+6x}\right]$

d) Evaluate $\lim_{x\to 2}\left[\frac{x^4-8x}{\sqrt{x^2+5}-3}\right]$

e) Evaluate $\lim_{x\to 0}\left[\frac{\cos 4x-\cos 6x}{x^2}\right]$

f) Evaluate $\lim_{x\to 0}\left[\frac{6^x-3^x-2^x+1}{x^2}\right]$

Q.4 Attempt any FOUR of the following **16 Marks**

a) Using first principle find derivative of $f(x)=\sin x$.

b) If u and v are differentiable functions of x and $y=u.v$, than prove that

$$\frac{dy}{dx}=u.\frac{dv}{dx}+v.\frac{du}{dx}$$

c) If $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$. find $\frac{dy}{dx}$

d) If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$

e) Find the derivative of $(\sin x)^{\cos x}$

f) If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Q.5 Attempt any FOUR of the following **16 Marks**

a) Evaluate $\lim_{x\to 0}\left[\frac{\sin 2x.\tan 4x.\log(1+x^2)}{x^4}\right]$

b) Evaluate $\lim_{x\to 3}\left[\frac{\log x - \log 3}{x-3}\right]$

c) Using Bisection method find the approximate root of $x^3 - x - 4 = 0$ [Carry out three iterations only].

d) Find the approximate root of the equation $x^3 - x - 1 = 0$ by using Regula false position method (Carry out three iterations only)

e) By using Newton- Raphson method find the positive root of $x^2 + x - 5 = 0$ correct to three decimal places.

f) Using Newton- Raphson method find approximate value of $\sqrt[3]{100}$, perform three iterations.

Q.6 Attempt any FOUR of the following **16 Marks**

a) If $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$.

b) If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

c) Solve the following equations by Gauss elimination method
$4x + y + 2z = 12$, $-x + 11y + 4z = 33$, $2x - 3y + 8z = 20$

d) Solve the following equations by Jacobi's method, by performing three iterations only
$10x + y + 2z = 13$, $3x + 10y + z = 14$, $2x+3y + 10z = 15$

e) Solve the following equations by Gauss- Seidal method taking two iterations
$10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$

f) With the following system of equation $5x - y = 9$, $5y - z = 6$, $x + 5z = -3$.
Set up the Gauss- Seidal iterations scheme for solution. Iterate two times, using initial approximations $x_0 = 1.8$, $y_o = 1.2$, $z_0 = -0.96$.